## Role of electronic correlations in the Fermi surface formation of Na<sub>x</sub>CoO<sub>2</sub>

A.O. Shorikov and V.I. Anisimov Institute of Metal Physics, Russian Academy of Sciences, 620219 Yekaterinburg GSP-170, Russia

## M.M. Korshunov

L.V. Kirensky Institute of Physics, Siberian Branch of Russian Academy of Sciences, 660036 Krasnoyarsk, Russia and
Max-Planck-Institut für Physik komplexer Systeme, D-01187 Dresden, Germany

(Dated: February 1, 2008)

Band structure of metallic sodium cobaltate  $Na_xCO_2$  (x=0.33, 0.48, 0.61 0.72) has been investigated by local density approximation+Hubbard U (LDA+U) method and within Gutzwiller approximation for the Co- $t_{2g}$  manifold. Correlation effects being taken into account results in suppression of the  $e'_g$  hole pockets at the Fermi surface in agreement with recent angle-resolved photo-emission spectroscopy (ARPES) experiments. In the Gutzwiller approximation the bilayer splitting is significantly reduced due to the correlation effects. The formation of high spin (HS) state in Co d-shell was shown to be very improbable.

PACS numbers: 74.70.-b; 31.15.Ar; 71.45.Gm; 71.10.-w

Introduction. Puzzling properties of sodium cobaltate Na<sub>x</sub>CO<sub>2</sub> are the topic of many recent theoretical and experimental investigations. This material holds much promise for thermoelectronics due to its large thermopower<sup>1</sup> together with the relatively low resistivity<sup>2</sup>. The discovery of superconductivity with  $T_c$  about 5K in Na<sub>0.33</sub>CoO<sub>2</sub>·1.3H<sub>2</sub>O<sup>2</sup> revived the interest in lamellar sodium cobaltates. Moreover, the charge and magnetic long range orders on the frustrated triangular lattice of cobaltate is of the fundamental interest. The band theory predict the complicated Fermi surface (FS) with one large hole pocket around the  $\Gamma = (0,0,0)$  point and six small pockets near the K =  $(0, \frac{4\pi}{3}, 0)$  points of the hexagonal Brillouin zone at least for  $x < 0.5^{3,4}$ . However, intensive investigations by several ARPES groups reveal absence of six small pockets in both Na<sub>x</sub>CO<sub>2</sub>·yH<sub>2</sub>O and in its parent compound  $Na_xCO_2^{5,6,7,8,9}$ .

The disagreement between ARPES spectra and abinitio calculated band structure points to the importance
of the electronic correlations in these oxides. Other evidences for the correlated behavior come from the data on
an anomalous Hall effect and a drop of the thermopower
in holistic magnetic field<sup>10</sup>.

The six hole pockets are absent in the L(S)DA+U calculations  $^{12,13}$ . However, in this approach, the insulating gap is formed by a splitting of the local single-electron states due to spin-polarization, resulting in a spin polarized Fermi surface with an area twice as large as that observed through ARPES. Moreover, the long range ferromagnetic order has been set by hand because of limitation of LDA+U. The predicted large local magnetic moments as well as the splitting of bands can be considered as artifacts of the L(S)DA+U method.

Although LDA+U method is usually applied to describe insulators, there are some achievement in investigation of metals and metallic compounds  $^{14,15}$ . To analyze the effect of electronic correlations on the Fermi surface formation in sodium cobaltate we employ both LDA+U method and a Gutzwiller approximation.

Co d-level splits by crystal field of oxygen octahedron in lower  $t_{2g}$  and higher  $e_g$  bands. The deficiency of Na in  $Na_xCO_2$  introduces additional holes in the system. Cobalt, having  $d^6$  configuration and filled  $t_{2g}$  shell in parent NaCO2, is nonmagnetic. But in nonstoichiometric compound part of Co ions become magnetic with local moment about  $1\mu_B$ . This value is provided by  $d^5$ configuration and one hole in  $t_{2q}$  shell. However, the experiments revealed the magnetic susceptibility at room temperature that is much higher than it was expected for dilute magnetic impurity in non-magnetic solvent. Explanation of this anomaly was suggested in Ref<sup>11</sup> as transition from low-spin (LS) state with six d-electrons on  $t_{2q}$  shell to high-spin (HS) state with five d-electrons on  $t_{2g}$  shell and one electron on  $e_g$ . The possibility of such transition will be discussed below.

LDA+U results. Na<sub>0.61</sub>CoO<sub>2</sub> crystallize in the hexagonal unit cell  $(P6_3/mmc$  space group) with a=2.83176(3)Å and c=10.8431(2)Å at 12 K Ref<sup>18</sup>. Displacement of Na atoms from their ideal sites 2d (1/3, 2/3, 3/4) on about  $0.2\mathring{A}$  are observed in nonstoichiometric cobaltates for both room and low temperatures. This is probably due to the repulsion of a randomly distributed Na atoms, locally violating hexagonal symmetry<sup>18</sup>. In the present investigation Na atoms are shifted back to their 2d ideal sites. In order to avoid charge disproportionation which can arises from some Na distribution if the supercell is used in calculation the change in the Na concentration has been considered in virtual crystal approximation (VCA) where each 2d site is occuped by virtual atoms with fractional number of valence electrons x and a core charge 11 - x instead of Na. Note, that all core states of virtual atom are left unchanged and corresponds to Na ones. We have chosen 4s, 4p, and 3d states of cobalt, 2s, 2p, and 3d states of oxygen, and 3s, 3p, and 3d states of Na as the valence states for TB-LMTO-ASA computation scheme. The radii of atomic spheres where 1.99 a.u. for Co, 1.61 a.u. for oxygen, and 2.68 a.u. for Na. Two classes of empty spheres

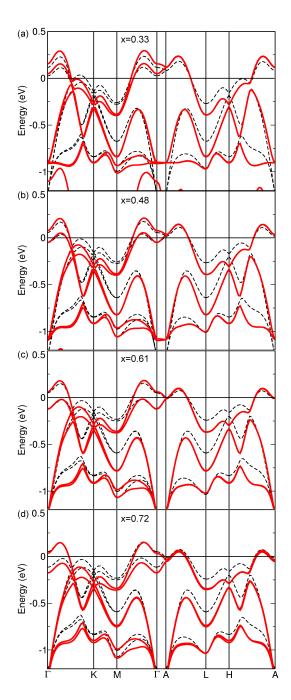


FIG. 1: (Color online) Band structure of  $Na_xCoO_2$  for x equal to 0.33 (a), 0.48 (b), 0.61 (c), and 0.72 (d), obtained within LDA is shown by the black (dashed) curves. Band structures for the same doping concentrations within LDA+U are shown by the red (solid) curves.

(pseudo-atoms without core states) were added to fill the unit cell volume.

Crystal field of oxygen octahedron splits Co d-band into 2-times degenerate  $e_g$  and 3-times degenerate  $t_{2g}$  subbands (without taking into account spin). LDA calculations shows that those manifolds are separated on about 2 eV<sup>3</sup>. Here partially filled  $t_{2g}$  subband crosses the Fermi level whereas  $e_g$  subband due to strongly hy-

bridization with nearest oxygen atoms is positioned well above the Fermi level. The procedure proposed in Ref. 19 allows one to calculate Coulomb repulsion parameter U taking into account the screening of localized d-shell by itinerant s- and p-electrons. Resulting U is equal to 6.7 eV. However, the presence of the  $t_{2g}$ - $e_g$  splitting give the reason to take into account an additional screening channel provided by the less localized  $e_g$  electrons. The value of U for  $t_{2g}$  orbitals calculated using "constrained LDA" method<sup>20</sup>, in which the screening by  $e_g$  electrons is also taken into account is equal to U=2.67 eV. The later value was used in the present calculation for all doping concentrations x. Hund's exchange parameter J depends weakly on screening effects due to its "on-site" character. Its value was calculated within "constrained LDA" method and is equal to 1.07 eV.

First, we have verified the possibility of HS state formation on Co d-shell. For this purpose the unit cell of Na<sub>0.61</sub>CO<sub>2</sub> with two Co atoms was considered. We have started from a saturated A-type antiferromagnetic configuration with five electrons on the  $t_{2g}$  and one on the  $e_g$ shells. Small U=2.67 eV does not stabilize such magnetic configuration and LS state was obtained. Increasing U up to 5 eV however results in HS state with large local magnetic moment about 1.96  $\mu_B$ . Nevertheless, this HS state has the total energy about 1.75 eV higher then the energy of a LS state. This large difference in total energy of both considered spin states arises form hexagonal structure of cobaltates where the angle of Co-O-Co bond is close to 90° in contrast to almost 180° for, e.g., RCoO<sub>3</sub> (R=La, Ho). In the later case the  $e_g$  band has the width about 3-5 eV and its bottom lies just above the Fermi level. The system wins energy of 2J forming a HS state overcoming the gap energy which is less than 1 eV. Due to this fact difference between LS and intermediate spin states in  $RCoO_3$  is less then 250 meV<sup>21</sup>. The angle of Co-O-Co bond is close to  $90^{\circ}$  in cobaltates and results in a weak overlap between  $e_g$  orbitals and hence in a narrow  $e_q$  band with larger gap between it and the  $t_{2q}$  band. Our calculation confirms that formation of HS in  $Na_xCoO_2$  is rather improbable and cannot be stabilized by any distortion of crystal structure or clusterization proposed in Ref.<sup>11</sup>. Local magnetic moments on Co sites can arise only due to holes doping due to Na atoms deficiency. Those holes order on Co atoms and form nonmagnetic  $\text{Co}^{3+}$  and magnetic  $\text{Co}^{4+}$  ions with  $d^6$  and  $d^5$  configurations, respectively. In the further calculations only the LS state was considered.

The ordering of holes on  $t_{2g}$  shell and corresponding long-range magnetic and charge order for Na<sub>0.5</sub>CoO<sub>2</sub> arise probably due to specific arrangement of Na atoms. This arrangements were observed experimentally<sup>22</sup> for several doping concentrations including x=0.5. Proper description of such order within "unrestricted Hartree-Fock" gives strong spin and orbital polarization and local magnetic moment of about  $1\mu_B$  on Co<sup>4+</sup> sites as well as insulating ground state with sizable gap. To describe non-ordered systems the implementation of "re-

stricted Hartree-Fock" method is more suitable. In the later starting from non-magnetic configuration of d-shell with equal number of spin-up and spin-down electrons LDA+U method gives non-magnetic solution without spin or orbital polarization. Note, that the gap does not open and Na $_x$ CoO $_2$  remains metallic for all Na concentration.

Obtained band structure of  $Na_xCoO_2$  for x=0.33, 0.48, 0.61, and 0.72 are shown in Fig. 1. Dashed (black) lines correspond to LDA results whereas solid (red) lines are the bands obtained by LDA+U method. Cobalt d and oxygen p states are separated by a small gap of about -1.25 eV for x=0.61 and x=0.72. However, this gap disappears for lower doping concentration since the d band goes down when the number of d electrons decreases. The presence of the two  $CoO_2$  layers within a unit cell due to alternation of the oxygen arrangement results in a bonding-antibonding (bilayer) splitting, also present in Fig. 1.

The degeneracy of the  $t_{2g}$  levels is partially lifted by the trigonal crystal field distortion which splits the former into the higher lying  $a_{1g}$  singlet and the lower two  $e_q'$  states. However, slight difference in occupation numbers of  $a_{1g}$  and  $e'_{q}$  orbitals (0.714 and 0.886 respectively for x=0.33) results in significant difference in LDA+U band structure in respect to LDA one. The energy of the less occupied  $a_{1g}$  orbital increases for both spins, whereas all  $e_g'$  bands go down (the total  $a_{1g}-e_g'$  splitting becomes 0.21 eV for x=0.33). This makes six  $e_g'$  Fermi surface hole pockets to disappear for small x values. Note, for all Na concentration LDA+U predicts large  $a_{1q}$  pocket of Fermi surface centered around the  $\Gamma$  point in excellent agreement with ARPES spectra for x < 0.7. The additional electron pocket close to the  $\Gamma$  point appears in both LDA and LDA+U methods for a large doping concentrations. It was discussed in our previous work $^{23,24}$ in connection with the electronic theory for the itinerant magnetism of this compound.

Gutzwiller approximation. The free electron Hamiltonian for  $CoO_2$ -plane in a hole representation is given by:

$$H_0 = -\sum_{\mathbf{f},\alpha,\sigma} (\varepsilon^{\alpha} - \mu) n_{\mathbf{f}\alpha\sigma} - \sum_{\mathbf{fg},\sigma} \sum_{\alpha,\beta} t_{\mathbf{fg}}^{\alpha\beta} d_{\mathbf{f}\alpha\sigma}^{\dagger} d_{\mathbf{g}\beta\sigma}. \quad (1)$$

where  $d_{\mathbf{f}\alpha\sigma}$   $(d^{\dagger}_{\mathbf{f}\alpha\sigma})$  is the annihilation (creation) operator for the  $t_{2g}$  hole at Co site  $\mathbf{f}$ , spin  $\sigma$  and orbital index  $\alpha$ ,  $n_{\mathbf{f}\alpha\sigma}=d^{\dagger}_{\mathbf{f}\alpha\sigma}d_{\mathbf{f}\alpha\sigma}$ , and  $t^{\alpha\beta}_{\mathbf{fg}}$  is the hopping matrix element,  $\varepsilon^{\alpha}$  is the single-electron energies, and  $\mu$  is the chemical potential. All in-plane hoppings and single-electron energies were derived previously<sup>23,24</sup> from the *ab-initio* LDA calculations. Since LDA-calculated hoppings and single-electron energies do not depend much on doping concentration, we used here parameters for Na<sub>0.33</sub>CoO<sub>2</sub> form Table I of Ref.<sup>24</sup>. To take bilayer splitting into account, we also consider here the hoppings  $t_z^{\alpha\beta}$  between adjacent CoO<sub>2</sub> planes. Their values were also derived from LDA results and are equal to  $t_z^{a_{1g}a_{1g}} = -0.0121$ ,  $t_z^{e'_{1g}e'_{g1}} = 0.0080$ , and  $t_z^{e'_{2g}e'_{g2}} = -0.0086$ .

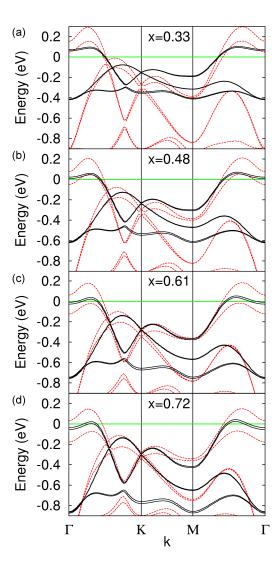


FIG. 2: (Color online) Band structure of  $Na_xCoO_2$  for x equal to 0.33 (a), 0.48 (b), 0.61 (c), and 0.72 (d), obtained in LDA+U is shown by the dashed (red) curves. Dispersion within the Gutzwiller approximation is shown by the solid (black) curves.

The Gutzwiller approximation  $^{25,26,27}$  for the Hubbard model provides a good description for the correlated metallic system. Its multiband generalization was formulated in Ref.  $^{28}$ . In this approach, the Hamiltonian describing the interacting system far from the metalinsulator transition for  $U \gg W$ , J=0:

$$H = H_0 + \sum_{\mathbf{f},\alpha} U_{\alpha} n_{\mathbf{f}\alpha\uparrow} n_{\mathbf{f}\alpha\downarrow}, \tag{2}$$

is replaced by the effective non-interacting Hamiltonian:

$$H_{eff} = -\sum_{\mathbf{f},\alpha,\sigma} (\varepsilon^{\alpha} + \delta \varepsilon^{\alpha\sigma} - \mu) n_{\mathbf{f}\alpha\sigma}$$
$$-\sum_{\mathbf{f}\neq\mathbf{g},\sigma} \sum_{\alpha,\beta} \tilde{t}_{\mathbf{f}\mathbf{g}}^{\alpha\beta} d_{\mathbf{f}\alpha\sigma}^{\dagger} d_{\mathbf{g}\beta\sigma} + C. \tag{3}$$

Here,  $\tilde{t}_{\mathbf{fg}}^{\alpha\beta}=t_{\mathbf{fg}}^{\alpha\beta}\sqrt{q_{\alpha\sigma}}\sqrt{q_{\beta\sigma}}$  is the renormalized hopping between two lattice sites connected by the spatial vector the orbital's filling factors,  $x = 1 - \sum_{\alpha\sigma} n_{\alpha\sigma} |\Psi_0\rangle \equiv \langle n_{\mathbf{f}\alpha\sigma} \rangle_0$  is the orbital's filling factors,  $x = 1 - \sum_{\alpha\sigma} n_{\alpha\sigma}$  is the equation for the chemical potential.  $\delta \varepsilon^{\alpha \sigma}$  are the Lagrange multipliers yielding the correlation induced shifts of the singleelectron energies. The constant  $C = \sum \delta \varepsilon^{\alpha\sigma} n_{\alpha\sigma}$  was

determined from the condition that the ground state energy is the same for both Hamiltonians,  $\langle \Psi_0 | H_{eff} | \Psi_0 \rangle =$  $\langle \Psi_q | H | \Psi_q \rangle$ , where  $| \Psi_0 \rangle$  is the wave function of the free electron system (3), and  $|\Psi_q\rangle$  is the Gutzwiller wave function for the Hamiltonian (2). The Lagrange multipliers are determined by minimizing the energy  $\langle \Psi_0 | H_{eff} | \Psi_0 \rangle$ with respect to the orbital filling factors  $n_{\alpha\sigma}$ . This results in the following expression for the single-electron energies renormalization:

$$\delta \varepsilon^{\alpha \sigma} = \frac{1}{2 (1 - n_{\alpha \sigma})} \sum_{\mathbf{f} \neq \mathbf{g}, \beta} \tilde{t}_{\mathbf{f} \mathbf{g}}^{\alpha \beta} \left\langle d_{\mathbf{f} \alpha \sigma}^{\dagger} d_{\mathbf{g} \beta \sigma} \right\rangle_{0}. \tag{4}$$

It is this energy shift that forces the  $e'_q$  bands to sink below the Fermi energy<sup>29</sup>. This is clearly seen in the doping-dependent evolution of the quasiparticle dispersion within Gutzwiller approximation in Fig. 2. To obtain these figures we solved self-consistently Eq. (4) and the equation on the chemical potential  $\mu$ .

The comparison of the Gutzwiller approximation results with the LDA+U dispersion reveals few very interesting conclusions. First, both approximations result in a suppression of  $e'_q$  hole pockets of the FS. Second, the bilayer splitting is strongly doping dependent and significantly reduced for Gutzwiller quasiparticles in comparison with the LDA+U quasiparticles because the renormalization coefficient,  $\sqrt{q_{\alpha\sigma}}\sqrt{q_{\beta\sigma}}$ , occurs not only for the in-plane hoppings, but also for the interlayer hoppings  $t_{z}^{\alpha\beta}$ . Third, when both bonding and antibonding  $t_{2q}$ bands do not cross the Fermi level around the  $\Gamma$  point, the FS crossings are the same in both approximations (see Fig. 2a). It is the simple consequence of the Luttinger theorem which holds for both approaches. But for large  $\boldsymbol{x}$  due to the larger bilayer splitting in LDA+ $\!U$  the Fermi

surfaces become different, while the Luttinger theorem is still preserved. With increase of the doping concentration x the bandwidth of the Gutzwiller quasiparticles becomes closer to the LDA+ ${\cal U}$  ones because band renormalization factor  $\sqrt{q_{\alpha\sigma}}\sqrt{q_{\beta\sigma}}$  comes closer to unity.

Dynamical Mean Field Theory (DMFT) calculations  $^{31,32}$  show that for the small U and non-zero  $J, e'_q$  FS pockets can by stabilized. On the other hand, DMFT calculations of Ref.<sup>30</sup> confirms results of the Gutzwiller approximation provided that U and the crystal filed slitting are large.

Conclusion. In the present work employing ab-initio "constrained LDA" method we obtained Coulomb repulsion parameter U=2.67 eV for  $t_{2g}$  orbitals taking into account the screening by  $e_q$ -electrons in addition to the screening by itinerant s- and p-electrons. Hund's exchnage parameter was found equal to J=1.07 eV.

Also, we have shown that due to the Co-O-Co bond angle close to  $90^{\circ}$  in  $Na_{x}CoO_{2}$  the energy gap between LS and HS states is too large to be overcome by clusterization or reasonable distortions of crystal structure. Thus we conclude that realization of HS state is highly improbable in these particular substance.

Then, to analyze the effect of electronic correlations on the Fermi surface topology of  $Na_xCoO_2$  we use two approaches, non-magnetic LDA+U and Gutzwiller approximation for the Hubbard-type model based on the LDA band structure. Within LDA+U the energy of the less occupied  $a_{1g}$  orbital increases for both spins, whereas all  $e'_q$  bands go down. This makes six  $e'_q$  FS hole pockets to disappear for small x values, in agreement with ARPES for x < 0.7. Gutzwiller approximation also resulted in a suppression of  $e'_a$  hole pockets at the FS. Moreover, the bilayer splitting was found to be strongly doping dependent and significantly reduced for Gutzwiller quasiparticles in comparison with LDA+U quasiparticles.

Acknowledgments. Authors thank I. Eremin and S.G. Ovchinnikov for useful discussions. A.S. and V.I.A. acknowledge the financial support from RFBR (Grants 06-02-81017, 07-02-00041). M.M.K. acknowledge support form INTAS (YS Grant 05-109-4891) and RFBR (Grants 06-02-16100, 06-02-90537-BNTS).

<sup>&</sup>lt;sup>1</sup> K. Fujita et al., Jpn. J. Appl. Phys. **40**, 4644 (2001).

<sup>&</sup>lt;sup>2</sup> K. Takada *et al.*, Nature (London), **422**, 53 (2003).

<sup>&</sup>lt;sup>3</sup> D.J. Singh, Phys. Rev. B **61**, 13397 (2000).

K.-W. Lee, et al., Phys. Rev. B 70, 045104 (2004).

M.Z. Hasan et al., Phys. Rev. Lett. 92, 246402 (2004).

<sup>&</sup>lt;sup>6</sup> H.-B. Yang et al., Phys. Rev. Lett. **92**, 246403 (2004).

H.-B. Yang et al., Phys. Rev. Lett. 95, 146401 (2005).

D. Qian et al., Phys. Rev. Lett. 96, 046407 (2006).

D. Qian et al., Phys. Rev. Lett. 96, 216405 (2006).

<sup>&</sup>lt;sup>10</sup> Y. Wang et al., Nature(London) **423**, 425 (2003).

M. Daghofer et al., Phys. Rev. Lett. 96, 216406 (2006).

<sup>&</sup>lt;sup>12</sup> S. Zhang *et al.*, Phys. Rev. Lett. **93**, 236402 (2005).

<sup>&</sup>lt;sup>13</sup> L.-J. Zou, et al., Phys. Rev. B **69**, 132505 (2004).

<sup>&</sup>lt;sup>14</sup> I. Yang et al., Phys. Rev. Lett. 87, 216405 (2001).

A.G. Petukhov et al., Phys. Rev. B 67, 153106 (2003).

V.I. Anisimov et al., Phys. Rev. B 43, 7570 (1991).

O.K. Andersen et al., Phys. Rev. Lett., 53, 2571 (1984)

J.D. Jorgensen et al., Phys. Rev. B, 68, 2145171, (2003).

O. Gunnarsson et al., Phys. Rev. B 39, 1708 (1989).

W.E. Pickett et al., Phys. Rev. B 58, 1201 (1998).

I.A. Nekrasov et al., Phys. Rev. B 68, 235113 (2003).

H.W. Zandbergen et al., Phys. Rev. B 70, 024101 (2004).

M.M. Korshunov et al., JETP Lett. 84, 650 (2006).

M.M. Korshunov et al., Phys. Rev. B 75, 094511 (2007).

M.C. Gutzwiller, Phys. Rev. Lett. 10, 159 (1963); Phys. Rev. B **134A**, 923 (1964); *ibid* **137A**, 1726 (1965).

- F. Gebhard, Phys. Rev. B 41, 9452 (1990).
   G. Kotliar et al., Phys. Rev. Lett. 57, 1362 (1986).
   J. Büunemann et al., J.Phys.: Cond.Mat. 9, 7343 (1997).
   S. Zhou et al., Phys. Rev. Lett. 94, 206401 (2005).
- $^{30}$  C.A. Marianetti  $et~al.,~{\rm cond\text{-}mat/0612606}$  (Unpublished).  $^{31}$  H. Ishida  $et~al.,~{\rm Phys.~Rev.~Lett.~}$  **94**, 196401 (2005).  $^{32}$  C.A. Perroni  $et~al.,~{\rm Phys.~Rev.~B}$  **75**, 045125 (2007).